Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Mark the Correct alternative in the following: If in a $\triangle$ABC, tan A + tan B + tan C = 0, then cot A cot B cot C = **[1]**

a) 6 b) None of these

c) $\frac{1}{6}$ d) 1

2. The range of the function f(x) = |x - 1| is **[1]**

a) $[0, \infty)$ b) $(0, \infty)$

c) $(-\infty, 0)$ d) R

3. What is the probability of 5 Sundays in the month of December? **[1]**

a) $\frac{1}{7}$ b) $\frac{3}{7}$

c) $\frac{2}{7}$ d) $\frac{4}{7}$

4. $\lim\limits_{x \to \pi} \frac{\sin x}{x - \pi}$ is equal to **[1]**

a) -1 b) 1

c) 2 d) -2

5. The number of points on X-axis which are at a distance of c units (c < 3) from (2, 3) is **[1]**

a) 0 b) 2

c) 3 d) 1

6. Let R be set of points inside a rectangle of sides a and b (a, b > 1) with two sides along the positive direction of x-axis and y-axis. Then **[1]**

a) R = {(x, y) : $0 \leq x < a, 0 \leq y \leq b$}
b) R = {(x, y) : $0 \leq x \leq a, 0 \leq y \leq b$}
c) R = {(x, y) : $0 \leq x \leq a, 0 < y < b$}
d) R = {(x, y) : $0 < x < a, 0 < y < b$}

7. Mark the correct answer for $3i^{34} + 5i^{27} - 2i^{38} + 5i^{41}$ = ? **[1]**

a) 1
b) 10i
c) -4i
d) -1

8. If A is a finite set containing n distinct elements, then the number of relations on A is equal to **[1]**

a) n^2
b) 2n
c) 2×2
d) 2^{n^2}

9. The solution set for $|3x - 2| \leq \frac{1}{2}$ **[1]**

a) $[\frac{5}{6}, \frac{1}{2}]$
b) $[\frac{1}{2}, \frac{5}{6}]$
c) $[\frac{5}{6}, \frac{2}{3}]$
d) $[\frac{2}{3}, \frac{2}{3}]$

10. The value of tan x + $\tan\left(\frac{\pi}{3} + x\right)$ + $\tan\left(\frac{2\pi}{3} + x\right)$ is **[1]**

a) 3 tan 3x
b) tan 3x
c) cot 3x
d) 3 cot 3x

11. If A = {1, 3, 5, 7, 9, 11, 13, 15, 17} ,B = {2, 4, ... , 18} and N the set of natural numbers is the universal set, then $A' \cup (A \cup B) \cap B')$ is **[1]**

a) A
b) ϕ
c) N
d) B

12. The sum of the infinite geometric series $(\sqrt{2} + 1) + 1 + (\sqrt{2} - 1) + \ldots.\infty$ = ? **[1]**

a) $\frac{(3+2\sqrt{2})}{2}$
b) $\frac{(4+3\sqrt{2})}{2}$
c) $\frac{(3+\sqrt{2})}{2}$
d) $\frac{(2+3\sqrt{2})}{2}$

13. $(\sqrt{5} + 1)^4 + (\sqrt{5} - 1)^4$ is **[1]**

a) a negative integer
b) a rational number
c) a negative real number
d) an irrational number

14. Solve the system of inequalities $2x + 5 \leq 0$, $x - 3 \leq 0$. **[1]**

a) $x \geq \frac{5}{2}$
b) $x \geq -\frac{5}{2}$
c) $x \leq -\frac{5}{2}$
d) $x \leq \frac{5}{2}$

15. The set A = {x : x is a positive prime number less than 10} in the tabular form is **[1]**

a) {2, 3, 5, 7}
b) {3, 5, 7}
c) {1, 3, 5, 7, 9}
d) {1, 2, 3, 5, 7}

16. The radius of the circle whose arc of length 15π cm makes an angle of 3n/4 radian at the centre is **[1]**

a) 20 cm

b) $11\frac{1}{4}$ cm

c) 10 cm

d) $22\frac{1}{2}$ cm

17. The solution set of $x^2 + 2x + 2 = 0$ is **[1]**

a) {1 + i, - i}

b) {i, - i}

c) {-1 + i, -1 -i}

d) {1 + i, 1 - i}

18. Find r if ${}^{10}P_r = 2.{}^{9}P_r$ **[1]**

a) 3

b) 6

c) 4

d) 5

19. If a, b ∈ R and x ∈ N, then **[1]**

$(a + b)^n = {}^nc_0a^n + {}^nc_1a^{n-1}b + {}^nc_2a^{n-2}b^2 + {}^nc_nb^n$

Assertion (A): The no of terms in binomial expansion is (n + 1)

Reason (R): Sum of indices of a and b in each term is (n + 1)

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If each of the observations $x_1, x_2, ..., x_n$ is increased by a, where a is a negative or positive number, then the variance remains unchanged. **[1]**

Reason (R): Adding or subtracting a positive or negative number to (or from) each observation of a group does not affect the variance.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the values of a and b, if **[2]**

i. (2a - 5, 4) = (5, b + 6)
ii. (a - 3, b + 7) = (3, 7)

OR

If $f(x) = y = \frac{ax-b}{cx-a}$, then prove that f(y) = x

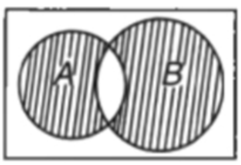
22. If $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$, show that $2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}$ **[2]**

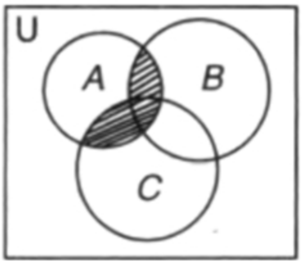
23. Find the equation of the ellipse the ends of whose major and minor axes are $(\pm 4, 0)$ and $(0, \pm 3)$ respectively. **[2]**

OR

Find the equation of the hyperbola whose conjugate axis is 5 and the distance between the foci is 13.

24. What is represented by the shaded regions in each of the following Venn-diagrams. **[2]**

i. 

ii. 

25. Find the equation of a line making an angle of 150° with the x-axis and cutting off an intercept 2 from y-axis. **[2]**

Section C

26. The Cartesian product $A \times A$ has 9 elements among which are found (-1, 0) and (0, 1). Find the set A and the remaining elements of $A \times A$. **[3]**

27. Solve the following system of inequations: **[3]**
$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$
$\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

28. Show that the points A(1, -1, -5), B(3, 1, 3) and C (9, 1, -3) are the vertices of an equilateral triangle. **[3]**

OR

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P(3, -2, 5).

29. Find the middle terms in the expansion of : $\left(\frac{x}{3} + 9y\right)^{10}$ **[3]**

OR

Find a if the coefficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

30. Solve the quadratic equation: $x^2 - (2 + i)x - (1 - 7i) = 0$. **[3]**

OR

If $(x + iy)^{1/3} = a + ib$, where x, y, a, b $\in$ R, then show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$.

31. There are 200 individuals with a skin disorder, 120 had been exposed to the chemical C_1, 50 to chemical C_2 and 30 to both the chemicals C_1 and C_2. Find the number of individuals exposed to (i) chemical C_1 but not chemical C_2 (ii) Chemical C_2 but not chemical C_1 (iii) Chemical C_2 or chemical C_1. **[3]**

Section D

32. A die is thrown. Find **[5]**

i. P(a prime number)
ii. P(a number $\geq$ 3)
iii. P(a number $\leq$ 1)
iv. P(a number more than 6)
v. P(a number less than 6)

33. Differentiate If y = $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$ show that $\frac{dy}{dx} = \sec x (\tan x + \sec x)$ **[5]**

OR

Evaluate the following limits: $\lim\limits_{x \to \sqrt{2}} \frac{\sqrt{3+2x}-(\sqrt{2}+1)}{x^2-2}$.

34. Find the sum of the following series up to n terms: **[5]**

i. 5 + 55 + 555 + ……
ii. 6 + .66 + .666 + …..

35. Find the value of $\cos^2 76^o + \cos^2 16^o - \cos 76^o \cos 16^o$. **[5]**

OR

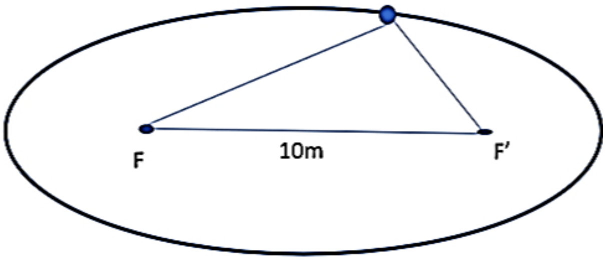
Prove that $\cot 7\frac{1^\circ}{2} = \tan 82\frac{1^\circ}{2} = (\sqrt{3}+\sqrt{2})(\sqrt{2}+1)$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



i. Name the curve traced by farmer and hence find the foci of curve. (1)
ii. Find the equation of curve traced by farmer. (1)
iii. Find the length of major axis, minor axis and eccentricity of curve along which farmer moves. (2)

OR

iv. Find the length of latus rectum. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Consider the data.

Class	Frequency
0-10	6
10-20	7
20-30	15
30-40	16
40-50	4
50-60	2

i. Find the mean deviation about median. (1)
ii. Find the Median. (1)
iii. Write the formula to calculate the Mean deviation about median? (2)

OR

Write the formula to calculate median? (2)

38. A permutation is **an act of arranging the objects or numbers in order**. Combinations are the way of selecting the objects or numbers from a group of objects or collections, in such a way that the order of the objects does not matter. **[4]**

ALLAHABAD

How many different words can be formed by using all the letters of the word ALLAHABAD?

i. In how many of them, vowels occupy the even position?

ii. In how many of them, both L do not come together?

Solution

Section A

1.

(b) None of these

Explanation:

Given ABC is a triangle, so $\angle$A + $\angle$B + $\angle$C = 180^o

Now applying tan on both sides

tan (A + B + C) = tan (180^o)

tan (A + B + C) = 0 ... (i)

Also given tan A + tan B + tan C = 0 ... (ii)

As per the formula of tan (A+B+C)

tan(A + B + C) = $\frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$

Now,

tan(A + B + C) = $\frac{0-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$ ... [from equation (i)]

0 = $\frac{-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$ ... [from equation (ii)]

By cross multiplying, we get

tan A tan B tan C = 0

$\implies \frac{1}{\tan A\tan B\tan C}$ = 0

Hence cot A cot B cot C = 0

2. **(a)** $[0, \infty)$

Explanation:

A modulus function always gives a positive value

R(f) =[$0, \infty$)

3.

(b) $\frac{3}{7}$

Explanation:

Number of days in December = 31

$\therefore$ Number of complete weeks = 4 (i.e. 7 $\times$ 4 = 28 days)

The remaining 3 days can be (M, T, W), (T, W, Th), (W, Th, F), (Th, F, Sa), (F, Sa, S), (Sa, S, M), (S, M, T)

Out of these 7, 3 are favourable outcomes.

So, the probability of having S Sunday in the month of December is $\frac{3}{7}$.

4. **(a)** -1

Explanation:

Given, $\lim_{x\to\pi}\frac{\sin x}{x-\pi}=\lim_{x\to\pi}\frac{\sin(\pi-x)}{-(\pi-x)}$

$=-1\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ and } \pi-x\to 0\Rightarrow x\to\pi\right]$

5. **(a)** 0

Explanation:

Let (x, 0) be the point.

Then, by distance formula we have

$(x - 2)^2 + 3^2 = c^2$

$x^2 - 4x + 4 + 9 - c^2 = 0$

$x^2 - 4x + 13 - c^2 = 0$

x will be real if the $b^2 - 4ac \geq 0$

i.e; 16 - 4(13 - c^2) ≥ 0

i.e; $c^2 - 9 \geq 0$

i.e; $|c| \geq 3$

Hence there is no point.

6.

(d) R = {(x, y) : 0 < x < a, 0 < y < b}

Explanation:

We have, R be set of points inside a rectangle of sides a and b

Since, a, b > 1

a and b cannot be equal to 0

Thus,R = {(x, y) : 0 < x < a, 0 < y < b}

7.

(d) -1

Explanation:

$3i^{34} + 5i^{27} - 2i^{38} + 5i^{41} = 3\times(i^4)^8\times i^2 + 5\times(i^4)^6\times i^3 - 2\times(i^4)^9\times i^2 + 5\times(i^4)^{10}\times i$

$= 3\times1\times(-1) + 5\times1\times(-i) - 2\times1\times(-1) + 5\times1\times i$

= -3 - 5i + 2 + 5i = -1

8.

(d) 2^{n^2}

Explanation:

The number of elements in A $\times$ A is n $\times$ n = n^2. Hence, the number of relations on A = number of subsets of A $\times$ A = $2^{n\times n}$ = 2^{n^2}.

9.

(b) $[\frac{1}{2}, \frac{5}{6}]$

Explanation:

$|3x - 2| \leq \frac{1}{2}$

$\Rightarrow \frac{-1}{2} \leq 3x - 2 \leq \frac{1}{2}$

$\Rightarrow \frac{-1}{2} + 2 \leq 3x - 2 + 2 \leq \frac{1}{2} + 2$

$\Rightarrow \frac{3}{2} \leq 3x \leq \frac{5}{2}$ $[\because |x| \leq a \Leftrightarrow -a \leq x \leq a\ ]$

$\Rightarrow \frac{3}{2} \cdot \frac{1}{3} \leq 3x \cdot \frac{1}{3} \leq \frac{5}{2} \cdot \frac{1}{3}$

$\Rightarrow \frac{1}{2} \leq x \leq \frac{5}{6}$

$\Rightarrow x \in \left[\frac{1}{2}, \frac{5}{6}\right]$

10. **(a)** 3 tan 3x

Explanation:

tan x + $\tan(60^\circ + x) + \tan(120^\circ + x)$ = $\tan x + \frac{\tan 60^\circ + \tan x}{1 - \tan 60^\circ \tan x} + \frac{\tan 120^\circ + \tan x}{1 - \tan 120^\circ \tan x}$

= $\tan x + \frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x} + \frac{(-\sqrt{3} + \tan x)}{1 + \sqrt{3}\tan x}$

= $\frac{\tan x\left(1 - 3\tan^2 x\right) + (\sqrt{3} + \tan x)(1 + \sqrt{3}\tan x) + (-\sqrt{3} + \tan x)(1 - \sqrt{3}\tan x)}{1 - 3\tan^2 x}$

= $\frac{\tan x - 3\tan^3 x + \sqrt{3} + 3\tan x + \tan x + \sqrt{3}\tan^2 x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1 - 3\tan^2 x}$

= $\frac{9\tan x - 3\tan^2 x}{1 - 3\tan^2 x}$

= $\frac{3\left(3\tan x - \tan^3 x\right)}{1 - 3\tan^2 x}$

= 3 tan 3x

11.

(c) N

Explanation:

We have,A′ ∪ (A ∪ B) ∩ B′

= A′ ∪ [(B’ ∩ A) ∪ (B’ ∩ B)] {∵ Distributive property of set: (A ∩ B) ∪ (A ∩ C) = A ∩ (B ∪ C)}

= A′ ∪ [(A ∩ B’) ∪ Φ] {∵ (B’ ∩ B) = ϕ}

= A′ ∪ (A ∩ B’) = (A’ ∪ A) ∩ (A’ ∪ B’) {∵ Distributive property of set: (A ∪ B) ∩ (A ∪ C) = A ∪ (B ∩ C)}

= Φ ∩ (A’ ∪ B’) {∵ (A’ ∩ A) = ϕ}

= (A’ ∪ B’) = (A ∩ B)’ {∵ (A’ ∪ B’)

= (A ∩ B)’} A′ ∪ (A ∪ B) ∩ B′ = (A ∩ B)’

A contains all odd numbers and B contains all even numbers

Therefore, A ∩ B = ϕ

⇒ A′ ∪ (A ∪ B) ∩ B′ = {ϕ}’

⇒ A′ ∪ (A ∪ B) ∩ B′ = N

12.

(b) $\frac{(4+3\sqrt{2})}{2}$

Explanation:

Given, a = $\left(\sqrt{2}+1\right)$ and r = $\frac{1}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} = \frac{(\sqrt{2}-1)}{1} = \left(\sqrt{2}-1\right)$ and |r| < 1.

$\therefore S_{\infty} = \frac{a}{(1-r)} = \frac{(\sqrt{2}+1)}{\{1-(\sqrt{2}-1)\}} = \frac{(\sqrt{2}+1)}{(2-\sqrt{2})} \times \frac{(2+\sqrt{2})}{(2+\sqrt{2})}$

$= \frac{4+3\sqrt{2}}{(4-2)} = \frac{4+3\sqrt{2}}{2}$.

13.

(b) a rational number

Explanation:

We have (a + b)^n + (a - b)^n

= $\left[{}^nC_0a^n +^n C_1a^{n-1}b +^n C_2a^{n-2}b^2 +^n C_3 \quad a^{n-3}b^3\right.$ + ... + $\left.{}^nC_nb^n\right]$ + $\left[{}^nC_0a^n -^n C_1a^{n-1}b +^n C_2a^{n-2}b^2 -^n C_3a^{n-3}b^3\right.$

+ ..... + $\left.(-1)^n \cdot^n C_n \quad b^n\right]$

= 2[$^nC_0 \quad a^n +^n C_2 \quad a^{n-2}b^2 +$ ...]

Let a = $\sqrt{5}$ and b = 1 and n = 4

Now we get $(\sqrt{5}+1)^4 + (\sqrt{5}-1)^4$ = 2 $\left[{}^4C_0(\sqrt{5})^4 +^4 C_2(\sqrt{5})^2 1^2 +^4 C_4(\sqrt{5})^0 1^4\right]$

= 2[25 + 30 + 1] = 112

14.

(c) $x \leq -\frac{5}{2}$

Explanation:

2x + 5 $\leq$ 0

⇒ 2x < -5

⇒ x $\leq \frac{-5}{2}$

⇒ $x \epsilon \left(-\infty, \frac{-5}{2}\right]$

Now x - 3 $\leq$ 0

⇒ x $\leq$ 3

⇒ $x \in (-\infty, 3]$

Hence the solution set is $\left(-\infty, \frac{-5}{2}\right] \cap (-\infty, 3]$ = $\left(-\infty, \frac{-5}{2}\right]$

⇒$x \leq -\frac{5}{2}$

15. **(a)** {2, 3, 5, 7}

Explanation:

Prime no. less then 10 is 2, 3, 5, 7 so

Set A = {2, 3, 5, 7}

16. **(a)** 20 cm

Explanation:

Here, arc length l = 15π cm

Angle $\theta = \frac{3\pi}{4}$

We know, angle subtended by the arc is given by,

$\theta = \frac{\text{length of arc}}{\text{radius}}$

$\therefore$ radius $= \frac{l}{\theta}$

$= \frac{15\pi}{3\pi} \times 4$

= 20 cm

17.

(c) {-1 + i, -1 -i}

Explanation:

$x^2 + 2x + 2 = 0 \Rightarrow$ x = $x = \frac{-2 \pm \sqrt{4-8}}{2}$

= $\frac{-2 \pm \sqrt{-4}}{2}$ = $\frac{-2 \pm 2i}{2}$

= $-1 \pm i$

$\therefore$ solution set = {-1 + i, -1 -i}

18.

(d) 5

Explanation:

Given ${}^{10}P_r = 2.{}^{9}P_r$

$\Rightarrow \frac{10!}{(10-r)!} = 2 \cdot \frac{(9)!}{(9-r)!}$

$\Rightarrow \frac{10 \times 9!}{(10-r) \times (9-r)!} = 2 \cdot \frac{(9)!}{(9-r)!}$

$\Rightarrow \frac{10}{(10-r)}$ = 2

$\Rightarrow$ 10 = 20 - 2r

$\Rightarrow$ 2r = 10

$\Rightarrow$ r = 5

19.

(c) A is true but R is false.

Explanation:

Assertion is true

$\because$ It is one of the observation of binomial expansion.

Reason:

Not true. AS sum of indices of a and b in each term is n.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Let $\bar{x}$ be the mean of x_1, x_2 ..., x_n. Then, variance is given by

$\sigma_1^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$

If a is added to each observation, the new observations will be

$y_i = x_i + a$

Let the mean of the new observations be $\bar{y}$.

Then,

$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{1}{n} \sum_{i=1}^{n} (x_i + a)$

$= \frac{1}{n} \left[\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} a \right]$

$= \frac{1}{n}\sum_{i=1}^{n} x_i + \frac{na}{n} = \bar{x} + a$

i.e. $\bar{y} = \bar{x} + a$ ...(ii)

Thus, the variance of the new observations is $\sigma_2^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i + a - \bar{x} - a)^2$ [using Eqs. (i) and (ii)]

$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \sigma_1^2$

Thus, the variance of the new observations is same as that of the original observations.

Reason: We may note that adding (or subtracting) a positive number to (or from) each observation of a group does not affect the variance.

Section B

21. We know that two ordered pairs are equal if their corresponding elements are equal.

i. $(2a-5, 4) = (5, b+6) \Rightarrow 2a - 5 = 5$ and $4 = b + 6$ [equating corresponding elements]

$\Rightarrow 2a = 5 + 5$ and $4 - 6 = b$

$\Rightarrow 2a = 10$ and $-2 = b \Rightarrow a = 5$ and $b = -2$

ii. $(a-3, b+7) = (3, 7) \Rightarrow$ a - 3 = 3 and b + 7 = 7 [equating corresponding elements]

$\Rightarrow$ a = 3 + 3 and b = 7 - 7 $\Rightarrow$ a = 6 and b = 0

OR

Here we have, f(x) = $y = \frac{ax-b}{cx-a}$

Therefore, $f(y) = \frac{ay-b}{cy-a} = \frac{a\left(\frac{ax-b}{cx-a}\right)-b}{c\left(\frac{ax-b}{cx-a}\right)-a}$

$= \frac{a(ax-b)-b(cx-a)}{c(ax-b)-a(cx-a)}$

$= \frac{a^2x-ab-bcx+ab}{acx-bc-acx+a^2} = \frac{a^2x-bcx}{a^2-bc} = \frac{x\left(a^2-bc\right)}{\left(a^2-bc\right)}$

$\therefore$ f(y) = x

Hence proved.

22. We have,

$y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

Differentiating with respect to x

$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$

Now,

LHS = $2x \cdot \frac{dy}{dx} + y$

LHS = $2x \times \left(\frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}\right) + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $\sqrt{x} - \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}}$

LHS = $2\sqrt{x}$

$\therefore$ LHS = RHS

Hence proved.

23. Given that:

End Points of Major Axis $= (\pm 4, 0)$ and End Points of Minor Axis $= (0, \pm 3)$

$\therefore$ The Equation of Ellipse is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ... (i)

where, a is the semi-major axis and b is the semi-minor axis.

So, a = 4 and b = 3

Putting the value of a and b in Eq. (i), we get

$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$

OR

Let 2a and 2b be the transverse and conjugate axes and e be the eccentricity. Let the centre be the origin and the transverse and the conjugate axes the coordinate axes. Then, the equation of the hyperbola is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

We have, 2b = 5 and 2 ae=13

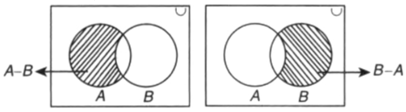
Now, $b^2 = a^2 (e^2 - 1)$

$\Rightarrow b^2 = a^2 e^2 - a^2$

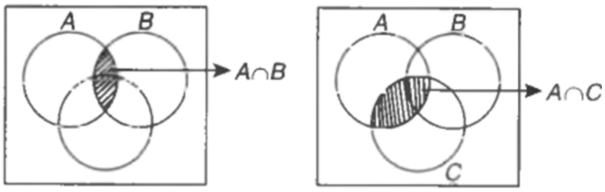
$\Rightarrow \frac{25}{4} = \frac{169}{4} - a^2 \Rightarrow a^2 = \frac{144}{4} \Rightarrow a = 6.$

Substituting the values of a and b in (i), the equation of the hyperbola is

$\frac{x^2}{36} - \frac{y^2}{25/4} = 1 \Rightarrow 25x^2 - 144y^2 = 900.$

24. 

$\therefore$ (A - B) $\cup$ (B - A)



$\therefore (A \cap B) \cup (A \cap C)$

or $A \cap (B \cup C)$

25. Given: m = tan 150° = -tan 30° = $-\frac{1}{\sqrt{3}}$ and c = y-intercept 2

Put the values of m and c in y = mx + c we obtain

$y = -\frac{1}{\sqrt{3}}x + 2$

$\Rightarrow x + \sqrt{3}y = 2\sqrt{3} = \frac{x}{\sqrt{3}} + \frac{y}{2} = 1$

Therefore, the equation of the required line is $x + \sqrt{3}y = 2\sqrt{3}$

Section C

26. Here $(-1, 0) \in A \times A \Rightarrow -1 \in A$ and $0 \in A$

$(0, 1) \in A \times A \Rightarrow 0 \in A$ and $1 \in A$

$\therefore -1, 0, 1, \in A$

It is given that $n(A \times A) = 9$ which implies that n (A) = 3

$\therefore$ A = (-1, 0, 1)

$\therefore A \times A$ = {(-1, -1), (-1, 0), (-1, 1) (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)}

So the remaining elements of $A \times A$ are

(-1, 1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0) and (1, 1)

27. The given system of inequation is

$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$ ...(i)

$\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$ ...(ii)

Now, $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$

$\Rightarrow \frac{10x+3x}{8} > \frac{39}{8}$

$\Rightarrow 13x > 39$

$\Rightarrow x > 3$

$\Rightarrow x \in (3, \infty)$

So, the solution set of inequation (i) is the interval $(3, \infty)$.

and, $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

$\Rightarrow \frac{(2x-1)-4(x-1)}{12} < \frac{3x+1}{4}$

$\Rightarrow \frac{-2x+3}{12} < \frac{3x+1}{4}$

$\Rightarrow -2x + 3 < 3(3x + 1)$ [Multiplying both sides by 12]

$\Rightarrow -2x + 3 < 9x + 3$

$\Rightarrow -2x - 9x < 3 - 3$

$\Rightarrow -11x < 0$

$\Rightarrow x > 0$

$\Rightarrow x \in (0, \infty)$

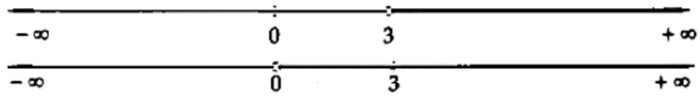
So, the solution set of inequation (ii) is the interval $(0, \infty)$.

These solution sets are graphed on the real line in Figure (i) and (ii) respectively.

From Fig. (i) and (ii), we observe that the intersection of the solution sets of inequations (i) and (ii) is the interval $(3, \infty)$ represented by the common thick line.

Hence, the solution set of the given system of inequations is the interval $(3, \infty)$.

i.
ii.



28. To prove: Points A, B, C form equilateral triangle.

Formula: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1, y_1, z_1)= (1, -1, -5)

(x_2, y_2, z_2)= (3, 1,3)

(x_3, y_3, z_3)= (9, 1, -3)

Length AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(3-1)^2 + (1-(-1))^2 + (3-(-5))^2}$

$= \sqrt{(2)^2 + (2)^2 + (8)^2}$

$= \sqrt{4+4+64}$

Length AB $= \sqrt{72} = 6\sqrt{2}$

Length BC = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(9-3)^2 + (1-1)^2 + (-3-3)^2}$

$= \sqrt{(6)^2 + (0)^2 + (-6)^2}$

$= \sqrt{36+0+36}$

Length BC $= \sqrt{72} = 6\sqrt{2}$

Length AC = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(9-1)^2 + (1-(-1))^2 + (-3-(-5))^2}$

$= \sqrt{(8)^2 + (2)^2 + (2)^2}$

$= \sqrt{64+4+4}$

Length AC $= \sqrt{72} = 6\sqrt{2}$

Hence, AB = BC = AC

Therefore, Points A, B, C make an equilateral triangle.

OR

Let Q (0, y, 0) be any point on y-axis.

$PQ = \sqrt{(0-3)^2 + (y+2)^2 + (0-5)^2}$

$= \sqrt{9 + y^2 + 4 + 4y + 25} = \sqrt{y^2 + 4y + 38}$

But $\sqrt{y^2 + 4y + 38} = 5\sqrt{2}$

Squaring both sides, we have

$y^2 + 4y + 38 = 50$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow (y - 2)(y + 6) = 0$

$\Rightarrow y = 2, -6$

Thus coordinates of point Q are (0, 2, 0) and (0, -6, 0)

29. Here n = 10, which is even.

So the middle term is $\left(\frac{10}{2} + 1\right)$ i.e. 6 th term

The general term in the expansion of $\left(\frac{x}{3} + 9y\right)^{10}$ is

$T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{10-r} \cdot (9y)^r \ldots$ (i)

Putting r = 5 in (i)

$T_6 = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} \cdot (9y)^5$

$= {}^{10}C_5 \frac{x^5}{3^5} \cdot 9^5 \cdot y^5 = 252 \times \frac{x^5}{243} \times 59049 y^5$

= 61236 x^5y^5

OR

Here $(3 + ax)^9 = {}^9C_0(3)^9 + {}^9C_1(3)^8(ax) + {}^9C_2(3)^7(ax)^2 + {}^9C_3(3)^6(ax)^3 + \ldots$

$= {}^9C_0(3)^9 + {}^9C_1(3)^8 \cdot a \cdot x + {}^9C_2(3)^7(a)^2 \cdot x^2 + {}^9C_3(3)^6 \cdot a^3x^3 + \ldots$

$\therefore$ Coefficient of $x^2 = {}^9C_2(3)^7a^2$

Coefficient of $x^3 = {}^9C_3(3)^6a^3$

It is given that

${}^9C_2(3)^7a^2 = {}^9C_3(3)^6a^3 \Rightarrow 36 \cdot 3^7a^2 = 84 \cdot 3^6 \cdot a^3$

$\Rightarrow a = \frac{36 \cdot 3^7}{84 \cdot 3^6} = \frac{108}{84} = \frac{9}{7}$.

30. $x^2 - (2 + i)x - (1 - 7i) = 0$

Comparing the given equation with the general form $ax^2 + bx + c = 0$, we get

a = 1, b = -(2 + i) and c = -(1 - 7i)

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ x = $\frac{(2+i) \pm \sqrt{(2+i)^2 + 4(1-7i)}}{2}$

$\Rightarrow$ x = $\frac{(2+i) \pm \sqrt{7-24i}}{2}$ .....(i)

Let x + iy = $\sqrt{7 - 24i}$. then,

$\Rightarrow (x + iy)^2 = 7 - 24i$

$\Rightarrow x^2 - y^2 + 2ixy = 7 - 24i$

$\Rightarrow x^2 - y^2 = 7$ .....(ii)

and 2xy = -24

Now, $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$\Rightarrow (x^2 + y^2)^2 = 49 + 576 = 625$

$\Rightarrow x^2 + y^2 = 25$ .....(iii)

add (ii) and (iii)

$2x^2$ =32 ==> $x^2 = 16$

$\Rightarrow x = \pm 4$

put value of x in (ii)

$y^2 = 9$

and $y = \pm 3$

As, xy is negative [From (ii)]

$\Rightarrow$ x = -4, y = 3 or, x = 4, y = -3

$\Rightarrow$ x + iy = -4 = 3i or, 4 - 3i

$\Rightarrow \sqrt{7 - 24i} = \pm 4 - 3i$

Substituting these values in (i), we get

$\Rightarrow$ x = $\frac{(2+i) \pm (4-3i)}{2}$

$\Rightarrow$ x = 3- i, -1 + 2i

So, the roots of the given quadratic equation are 3 - i and -1 + 2i.

OR

We have, $(x + iy)^{1/3} = a + ib$

$\Rightarrow x + iy = (a + ib)^3$ [cubing on both sides]

$\Rightarrow x + iy = a^3 + i^3 b^3 + 3 iab (a + ib)$

$\Rightarrow x + iy = a^3 - ib^3 + i\, 3a^2 b - 3ab^2$

$\Rightarrow x + iy = a^3 - 3ab^2 + i (3a^2 b - b^3)$

On equating real and imaginary parts from both sides, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

$\Rightarrow \frac{x}{a} = a^2 - 3b^2$ and $\frac{y}{b} = 3a^2 - b^2$

Now, $\frac{x}{a} - \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

$= -2a^2 - 2b^2 = -2(a^2 + b^2)$

Hence proved.

31. Let S denote the universal set consisting of individuals suffering from the skin disorder, A denote the set of individuals exposed to chemical C_1 and B denote the set of individuals exposed to chemical C_2.

Now,

n(S) = 200

n(A) = 120

n(B) = 50

and n (A ∩ B) = 30

i. Chemical C_1 but not chemical C_2

Number of individuals exposed to chemical C_1 but not chemical C_2 is

$= n(A \cap B')$

$= n(A) - n(A \cap B)$

= 120 - 30 = 90

ii. Number of individuals exposed to chemical C_2 but not chemical C_1

$= n(A' \cap B)$

= n(B) - n(A ∩ B)

= 50 - 30 = 20

iii. Number of individuals exposed to chemical C_1 or chemical C_2

$= n(A \cup B)$

= n(A) + n(B) - n(A ∩ B)

= 120 + 50 - 30

= 140

Section D

32. S= $\{1, 2, 3, 4, 5, 6\}$

$\therefore n(S) = 6$

i. Suppose A be the event of getting a prime number.

Then, $A = \{2, 3, 5\}$

$\therefore n(A) = 3$

Now, probability of event of getting prime number,

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

ii. Suppose event B = Getting a number $\geq$ 3. Then,

$B = \{3, 4, 5, 6\}$

$\therefore n(B) = 4$

Now, probability of event B,

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

iii. Suppose E_1 be the event of getting a number $\leq$ 1. Then,

$E_1 = \{1\}$

$\therefore n(E_1) = 1$

Now, probability of event E_1,

$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{6}$

iv. Suppose E_2 be the event of getting a number more than 6.

Then,

$E_2 = \{\ \} = \phi$

⇒ E_2 is an impossible event.

∴ Probability of event getting a number more than 6,

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{0}{6} = 0$

v. Suppose E_3 be the event of getting a number less than 6. Then,

$E_3 = \{1, 2, 3, 4, 5\}$

$\therefore n(E_3) = 5$

Now, probability of event E_3,

$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{5}{6}$

33. We have to show that $\frac{dy}{dx} =$ (sec x tan x + sec x)

where, it is given that

y = $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$

y $= \sqrt{\frac{\frac{1}{\cos x}\frac{\sin x}{1}}{\cos x} + \frac{\sin x}{\cos x}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$

u = 1 - sin x, v = 1 + sin x, $x = \frac{1-\sin x}{1+\sin x}$

if z = $\frac{u}{v}$

$\frac{dz}{dx} = \frac{\mathrm{v} \times \frac{\mathrm{du}}{\mathrm{dx}} - \mathrm{u} \times \frac{\mathrm{dv}}{\mathrm{dx}}}{\mathrm{v}^2}$

$= \frac{(1+\sin x) \times (-\cos x) - (1-\sin x) \times (\cos x)}{(1+\sin x)^2}$

$= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1+\sin x)^2}$

$= \frac{-2\cos x}{(1+\sin x)^2}$

According to the chain rule of differentiation

$\frac{dy}{dx} = \frac{\mathrm{dy}}{\mathrm{dz}} \times \frac{\mathrm{dz}}{\mathrm{dx}}$

$= \left[-\frac{\cos x}{1} \times \left(\frac{1-\sin x}{1}\right)^{-\frac{1}{2}}\right] \times \left[\frac{1}{(1+\sin x)^{2-\frac{1}{2}}}\right]$

$= \left[\cos x \times (1+\sin x)^{-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{3}{2}} \times \left(\frac{1+\sin x}{1+\sin x}\right)^{\frac{3}{2}}$

Multiplying and dividing by $(1+\sin x)^{\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{2}{2}} \times \left(\frac{1}{1+\sin x}\right)^{\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^{\frac{2}{2}-\frac{1}{2}}\right] \times (1-\sin x)^{-\frac{2}{2}} \times (1+\sin x)^{-\frac{2}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times \left(1-\sin^2 x\right)^{-\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times \left(\cos^2 x\right)^{-\frac{3}{2}}$

$= \left[\cos x \times (1+\sin x)^1\right] \times (\cos x)^{-3}$

$= \left[(1+\sin x)^1\right] \times (\cos x)^{-3+1}$

$= \frac{1+\sin x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} \times \frac{1+\sin x}{\cos^2 x}$

$= \sec x \left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)$

= sec x (sec x + tan x)

Hence proved

OR

We have to find the value $\lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - (\sqrt{2}+1)}{x^2-2}$

Re-writing the equation as

$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \sqrt{(\sqrt{2}+\sqrt{1})^2}}{x^2-2}$

$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \sqrt{2+1+2\sqrt{2}}}{x^2-2}$

$= \lim_{x \to \sqrt{2}} \frac{\sqrt{3+2x} - \sqrt{3+2\sqrt{2}}}{x^2-2}$

Now rationalizing the above equation

$= \lim_{x\to\sqrt{2}} \frac{(\sqrt{3+2x}-\sqrt{3+2\sqrt{2}})}{x^2-2} \frac{(\sqrt{3+2x}+\sqrt{3+2\sqrt{2}})}{(\sqrt{3+2x}+\sqrt{3+2\sqrt{2}})}$

Formula: $(a + b)(a - b) = a^2 - b^2$

$= \lim_{x\to\sqrt{2}} \frac{(3+2x-(3+2\sqrt{2}))}{x^2-2} \frac{(1)}{(\sqrt{3+2x}+\sqrt{3+2\sqrt{2}})}$

$= \lim_{x\to\sqrt{2}} \frac{(2x-2\sqrt{2})}{x^2-2} \frac{(1)}{(\sqrt{3+2x}+\sqrt{3+2\sqrt{2}})}$

$= \lim_{x\to\sqrt{2}} \frac{2(x-\sqrt{2})}{(x+\sqrt{2})(x-\sqrt{2})} \frac{(1)}{(\sqrt{3+2x}+\sqrt{3+2\sqrt{2}})}$

$= \frac{2}{2\sqrt{2}} \frac{1}{(2\sqrt{3+2\sqrt{2}})}$

$= \frac{1}{2\sqrt{2}} \frac{1}{(\sqrt{3+2\sqrt{2}})}$

34. i. S_n = 5 + 55 + 555 + ………. up to n terms

= 5 [1 + 11 + 111 + ………. up to n terms]

$= \frac{5}{9}$ [9 + 99 + 999 + ... up ton terms]

$= \frac{5}{9}$ [(10 - 1) + (10^2 - 1) + (10^3 - 1) + ... up to n terms]

$= \frac{5}{9} \left[\frac{10(10^n-1)}{10-1} - n\right]$

$= \frac{5}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$= \frac{50}{81}(10^n - 1) - \frac{5}{9}n$

ii. S_n = .6 + .66 + .666 + …………. up to n terms

= 6 [.1 + .11 + .111 + ………. up to n terms]

$= \frac{6}{9}$ [.9 + .99 + .999 + ... up to n terms]

$= \frac{6}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots\ldots \text{ up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) \ldots\ldots \text{ up to } n \text{ terms}\right]$

$= \frac{6}{9}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \ldots\ldots\ldots \text{ up to } n \text{ terms}\right)\right]$

$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}\right]$

$= \frac{2}{3}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^2}\right)\right]$

$= \frac{2n}{3} - \frac{2}{27}\left(1 - \frac{1}{10^n}\right)$

35. According to the question, we can write ,

$\cos^2 76^o + \cos^2 16^o - \cos 76^o \cos 16^o$

In the above expression consider $\cos 76^o \cos 16^o$

[By using the trigonometric sum formula, we can say that,

cos(C + D) + cos (C - D) = 2 cos C cos D]

Now multiply and divide this with 2, we get

$\frac{2\times(\cos 76^\circ \cos 16^\circ)}{2} = \frac{\cos(76^\circ+16^\circ)+\cos(76^\circ-16^\circ)}{2}$

$= \frac{\cos 92^\circ + \cos 60^\circ}{2}$

Consider the full expression,

$= \cos^2 76^o + \cos^2 76^o - \cos 76^o \cos 16^o$

$= \cos^2 76^o + \cos 16^o - \left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

Multiplying and dividing the terms $\cos^2 76° + \cos^2 16°$ with 2

$= \frac{2\cos^2 76^\circ}{2} + \frac{2\cos^2 16^\circ}{2} - \left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

$= \frac{1}{2}[\cos 2(76) + 1] + \frac{1}{2}[\cos 2(16) + 1] - \left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

[by using the formula, $\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow 2\cos^2\theta = \cos 2\theta + 1$]

$= \frac{1}{2}[2 + (\cos 152^o + \cos 32^o)] - \left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

[by using the formula, cos A + cos B = 2 cos$\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)$]

= 1 + $\frac{1}{2}$[2 cos$\left(\frac{152^\circ+32^\circ}{2}\right)$ cos$\left(\frac{152^\circ-32^\circ}{2}\right)$] - $\left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

= 1 + $\frac{1}{2}$[2 cos$\left(\frac{184^\circ}{2}\right)$ cos$\left(\frac{120^\circ}{2}\right)$] - $\left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

= 1 + $\frac{1}{2}$[2 cos(92^o) cos(60^o)] - $\left(\frac{\cos 92^\circ+\cos 60^\circ}{2}\right)$

= 1 + $\frac{\cos 92^\circ}{2}$ - $\frac{\cos 92^\circ}{2}$ - $\frac{\frac{1}{2}}{2}$

= 1 - $\frac{1}{4}$ = $\frac{3}{4}$

Hence, cos^2 76^o + cos^2 16^o - cos 76^o cos 16^o =$\frac{3}{4}$

Hence, the required value is calculated.

OR

LHS $= tan\ 82\frac{1^\circ}{2} = tan(90^o - 7\frac{1^\circ}{2}) = cot\ 7\frac{1^\circ}{2} = cotA$ [say]

where, $A = 7\frac{1^\circ}{2}$

Now, $cotA = \frac{\cos A}{\sin A} = \frac{\cos A(2\cos A)}{\sin A(2\cos A)}$

[multiplying numerator and denominator by $2cos\ A$]

= $\frac{2\cos^2 A}{2\sin A\cdot\cos A}$

= $\frac{1+\cos 2A}{\sin 2A}$ [$\because cos^2 x = \frac{1+\cos 2x}{2}$ and $sin\ 2x = 2sin\ x \times cos\ x$]

$\Rightarrow \cot 7\frac{1^\circ}{2} = \frac{1+\cos 2\left(7\frac{1^\circ}{2}\right)}{\sin 2\left(7\frac{1^\circ}{2}\right)} = \frac{1+\cos 2\left(\frac{15}{2}\right)^\circ}{\sin 2\left(\frac{15}{2}\right)^\circ}$ [put A = $7\frac{1^\circ}{2}$]

$\Rightarrow \cot 7\frac{1}{2} = \frac{1+\cos 15^\circ}{\sin 15^\circ} = \frac{1+\cos(45^\circ-30^\circ)}{\sin(45^\circ-30^\circ)}$

= $\frac{1+(\cos 45^\circ\cdot\cos 30^\circ+\sin 45^\circ\cdot\sin 30^\circ)}{(\sin 45^\circ\cdot\cos 30^\circ-\cos 45^\circ\cdot\sin 30^\circ)}$

[$\because cos(x-y) = cos\ x\ cosy + sin\ x\ sin\ y$ and $sin(x-y) = sin\ x\ cos\ y - cos\ x\ sin\ y$]

= $\frac{1+\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\cdot\frac{1}{2}\right)}$

= $\frac{1+\left(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\right)}$

= $\frac{2\sqrt{2}+\sqrt{3}+1}{(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

[multiplying numerator and denominator by $\sqrt{3}$ + 1]

= $\frac{2\sqrt{2}(\sqrt{3}+1)+(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

= $\frac{2\sqrt{6}+2\sqrt{2}+(\sqrt{3}+1)^2}{3-1} = \frac{2\sqrt{6}+2\sqrt{2}+3+1+2\sqrt{3}}{2}$

= $\frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2}$

= $\frac{2(\sqrt{6}+\sqrt{2}+2+\sqrt{3})}{2} = \sqrt{6}+\sqrt{2}+2+\sqrt{3}$

= $\sqrt{2}\cdot\sqrt{3}+\sqrt{2}+\sqrt{2}\cdot\sqrt{2}+\sqrt{3}$

= $(\sqrt{2}\cdot\sqrt{3}+\sqrt{2}\cdot\sqrt{2})+(\sqrt{2}+\sqrt{3})$

= $\sqrt{2}(\sqrt{3}+\sqrt{2})+1(\sqrt{2}+\sqrt{3}) = (\sqrt{2}+1)(\sqrt{3}+\sqrt{2})$

= RHS

$\therefore$ **LHS = RHS**

Hence proved.

Section E

36. i. The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Two positions of hand pumps are foci Distance between two foci = 2c = 10 Hence c = 5 Here foci lie on x axis & coordinates of foci = $(\pm c, 0)$

Hence coordinates of foci =$(\pm 5, 0)$

ii. $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci = 2a

Sum of distances between the farmer and each hand pump is = 26 = 2a

$\Rightarrow$ 2a = 26 $\Rightarrow$ a = 13 m

Distance between the handpump = 10m = 2c

$\Rightarrow$ c = 5 m

$c^2 = a^2 - b^2$

$\Rightarrow 25 = 169 - b^2$

$\Rightarrow b^2 = 144$

Equation is $\frac{x^2}{169}+\frac{y^2}{144}=1$

iii. Equation of ellipse is $\frac{x^2}{169}+\frac{y^2}{144}=1$ comparing with standard equation of ellipse a=13, b= 12 and c= 5 (given)

Length of major axis = 2a = 2 $\times$ 13 = 26

Length of minor axis = 2b = 2 $\times$ 12 = 24

eccentricity $e=\frac{c}{a}=\frac{5}{13}$

OR

Equation of the ellipse is $\frac{x^2}{169}+\frac{y^2}{144}=1$ hence a = 13 and b = 12

length of latus rectum of ellipse is given by $\frac{2\,b^2}{a}=\frac{2\times144}{13}$

37. i. We make the table from the given data.

Class	f_i	cf	Mid-point(x_i)	$\|x_i - M\|$	$f_i\|x_i - M\|$
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
	50				508

Here, $\frac{N}{2}$ = $\frac{50}{2}$ = 25

Here, 25th item lies in the class 20-30. Therefore, 20-30 is the median class.

Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l+\frac{\frac{N}{2}-cf}{f}\times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}$ $\times$ 10 = 20 + 8 = 28

Thus, mean deviation about median is given by

MD (M) = $\frac{1}{N}\sum_{i=1}^{6} f_i\,|x_i-M|$ = $\frac{1}{50}$ $\times$ 508 = 10.16

Hence, mean deviation about median is 10.16.

ii. Here, l = 20, cf = 13, f = 15, b = 10 and N = 50

$\because$ Median, M = $l+\frac{\frac{N}{2}-cf}{f}\times b$

$\Rightarrow$ M = 20 + $\frac{25-13}{15}$ $\times$ 10 = 20 + 8 = 28

iii. MD = $\frac{\Sigma f_i|x_i-M|}{N}$

OR

M = $1+\frac{\frac{N}{2}-cf}{f}\times h$

38. In a word ALLAHABAD, we have

Letters	A	L	H	B	D	Total
Number	4	2	1	1	1	9

So, the total number of words = $\frac{9!}{4!2!}=\frac{9\times8\times7\times6\times5}{2\times1}$ = 7560

i. There are 4 vowels and all are alike i.e., 4 A’s.

Also, there are 4 even places which are 2nd, 4th, 6th and 8th. So, these 4 even places can be occupied by 4 vowels in $\frac{4!}{4!}$ = 1 way. Now, we are left with 5 places in which 5 letters, of which two are alike (2 L’s) and other distinct, can be arranged in $\frac{5!}{2!}$ ways.

Hence, the total number of words in which vowels occupy the even places $= \frac{5!}{2!} \times \frac{4!}{4!} = \frac{5!}{2!}$ = 60

ii. Considering both L together and treating them as one letter. We have,

Letters	A	LL	H	B	D	Total
Number	4	1	1	1	1	8

Then, 8 letters can be arranged in $\frac{8!}{4!}$ ways.

So, the number of words in which both L come together = $\frac{8!}{4!}$ = 8 $\times$ 7 $\times$ 6 $\times$ 5 = 1680

Hence, the number of words in which both L do not come together

= Total number of words - Number of words in which both L come together

= 7560 - 1680 = 5880

Hence, the total number of words in which both L do not come together is 5880