Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. If $\cos\theta = \frac{-12}{13}$ and $\pi < \theta < \frac{3\pi}{2}$, then $\left(\cot\theta + \operatorname{cosec}\theta\right)$ = ? **[1]**

a) $\frac{1}{5}$ b) $\frac{-3}{5}$

c) $\frac{3}{5}$ d) $\frac{-1}{5}$

2. If R is a relation from a non – empty set A to a non – empty set B, then **[1]**

a) $R = A \cap B$ b) $R = A \cup B$

c) $R \subseteq A \times B$ d) $R = A \times B$

3. One card is drawn from a pack of 52 cards. The probability that it is the card of a king or spade is **[1]**

a) $\frac{3}{13}$ b) $\frac{1}{26}$

c) $\frac{4}{13}$ d) $\frac{3}{26}$

4. $\lim\limits_{x \to 0} \frac{(1+x)^n - 1}{x}$ is equal to **[1]**

a) -n b) n

c) 0 d) 1

5. The reflection of the point (α, β, γ) in the xy- plane is **[1]**

a) (0, 0, γ) b) (α, β, 0)

c) (α, β, -γ) d) (-α, -β, γ)

6. If the rth term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4 then r is equal to **[1]**

a) 3
b) 4
c) 1
d) 2

7. If z = cos $\frac{\pi}{4}$ + i sin $\frac{\pi}{6}$, then **[1]**

a) $|z|=1$, arg (z) = $\frac{\pi}{6}$
b) $|z|=1$, arg (z) = $\frac{\pi}{4}$
c) $|z|=\frac{\sqrt{3}}{2}$, arg (z) = $\tan^{-1}\frac{1}{\sqrt{2}}$
d) $|z|=\frac{\sqrt{3}}{2}$, arg (z) = $\frac{5\pi}{24}$

8. Let f(x) = $\sqrt{\log(2x-x^2)}$ Then, dom f(x) = ? **[1]**

a) [1, 2]
b) (0, 2)
c) none of these
d) $(-\infty, 1]$

9. If $x < 5$, then **[1]**

a) $x > -5$
b) $-x < 5$
c) none of these .
d) $-x > -5$

10. $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$ = ? **[1]**

a) $\frac{\sqrt{3}}{2}$
b) $\frac{(2+\sqrt{3})}{2}$
c) $\frac{\sqrt{3}+1}{2}$
d) $\frac{1}{2}$

11. If A = {1, 3, 5, B} and B = {2, 4}, then **[1]**

a) $\{4\} \subset A$
b) $4 \in A$
c) None of these
d) $B \subset A$

12. If the nth term of a GP is 2n, the sum of its first 6 terms is **[1]**

a) 190
b) 126
c) 124
d) 254

13. The 10th term in the expansion of $\left(2x^2+\frac{1}{x^2}\right)^{12}$ is **[1]**

a) $1760\ x^{-12}$
b) $792\ x^{-14}$
c) $264\ x^{-16}$
d) $692\ x^{-12}$

14. Find all pairs of consecutive even positive integers, both of which are larger than 5, such that their sum is less than 23. **[1]**

a) (6, 8),(8, 10), (10, 12)
b) (8, 6), (6, 4), (10, 6)
c) (3, 5), (5, 7), (7, 9)
d) (4, 6), (6, 8), (8, 10)

15. If A = {x : x $\neq$ x} represents **[1]**

a) {x}
b) { }
c) {1}
d) {0}

16. The value of tan 75° - cot 75° is equal to **[1]**

a) $2+\sqrt{3}$
b) $2-\sqrt{3}$

c) $2\sqrt{3}$ d) $1+2\sqrt{3}$

17. The multiplicative inverse of $(3 + 2i)^2$ is **[1]**

a) $\left(\frac{5}{169}+\frac{12}{169}i\right)$ b) $\left(\frac{-4}{169}+\frac{12}{169}i\right)$

c) $\left(\frac{5}{169}-\frac{12}{169}i\right)$ d) $\left(\frac{-5}{169}+\frac{12}{169}i\right)$

18. The number of six letter words that can be formed using the letters of the word **ASSIST** in which S's alternate with other letters is **[1]**

a) 18 b) 12

c) 14 d) 24

19. **Assertion (A):** if A = set of letters in **Alloy** B = set of letters in **LOYAL**, then set A & B are equal sets. **[1]**
Reason (R): If two sets have exactly the same elements, they are called equal sets.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. **[1]**
Reason (R): The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Determine the domain and range of the following relation **[2]**
R = {(x, y):x ∈ N, y ∈ N and x + y = 10}

OR

Find the values of a and b, when (a + 3, b - 2) = (5, 1)

22. If $\lim\limits_{x \to a} \frac{x^5-a^5}{x-a}$ = 405, find all possible values of a. **[2]**

23. Find the equation of the circle passing through the point (2, 4) and having its centre at the intersection of the lines x - y = 4 and 2x + 3 y + 7 = 0. **[2]**

OR

Find the vertex, focus, axis, directrix and latus-rectum of the following parabolas $y^2 - 4y + 4x = 0$

24. Is the pair of set A = {2, 3} and B = {x : x is solution of $x^2 + 5x + 6 = 0$} equal? Give reason. **[2]**

25. The slope of a line is double the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, then find the slope of the lines. **[2]**

Section C

26. Let X = {1, 2, 3, 4} and Y = {1, 4, 9, 16, 25}. **[3]**

Let f = {(x, y): x ∈ X, y ∈ Y and $y = x^2$}.

i. Show that f is a function from X to Y. Find its domain and range.
ii. Draw a pictorial representation of the above function.
iii. If A = [2,3, A], find f (A).

27. Solve the inequality $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$ for real x. **[3]**

28. Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available. **[3]**

OR

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and C (6, 0, 0).

29. If in the expansion of $(1 + x)^n$, the coefficients of three consecutive terms are 56, 70 and 56, then find n and the position of the terms of these coefficients. **[3]**

OR

Show that the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the coefficients of middle terms in the expansion of $(1 + x)^{2n-1}$.

30. Convert the complex number $\frac{1}{(1+i)}$ into polar form. **[3]**

OR

Solve $3x^2 - 4x + \frac{20}{3} = 0$

31. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find the number of people who read exactly one newspaper. **[3]**

Section D

32. A sample space consists of 9 elementary outcomes $e_1, e_2, ..., e_9$ whose probabilities are **[5]**

$P(e_1) = P(e_2) = .08, P(e_3) = P(e_4) = P(e_5) = 0.1$

$P(e_6) = P(e_7) = .2, P(e_8) = P(e_9) = .07$

Suppose $A = \{e_1, e_5, e_8\}$, $B = \{e_2, e_5, e_8, e_9\}$

i. Calculate P (A), P (B), and P (A ∩ B)
ii. Using the addition law of probability, calculate P (A ∪ B)
iii. List the composition of the event A ∪ B, and calculate P (A ∪ B) by adding the probabilities of the elementary outcomes.
iv. Calculate P ($\bar{B}$) from P ($\bar{B}$), also calculate P ($\bar{B}$) directly from the elementary outcomes of $\bar{B}$

33. Find the differential coefficient of sec x, using first principle. **[5]**

OR

Find the derivative of (sinx + cosx) from first principle.

34. The ratio of A M and G. M of two positive no. a and b is m : n show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$. **[5]**

35. Prove that $\cos 12^o + \cos 60^o + \cos 84^o = \cos 24^o + \cos 48^o$ **[5]**

OR

If $\sin \theta + \sin \phi = \sqrt{3} (\cos \phi - \cos \theta)$, then show that $\sin 3\theta + \sin 3\phi = 0$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Arun is running in a racecourse note that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m.

i. Path traced by Arun represents which type of curve. Find the length of major axis? (1)

ii. Find the equation of the curve traced by Arun? (1)

iii. Find the eccentricity of path traced by Arun? (2)

OR

iv. Find the length of latus rectum for the path traced by Arun. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15 , respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hindi	S.St	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

i. Find the correct variance. (1)

ii. What is the formula of variance. (1)

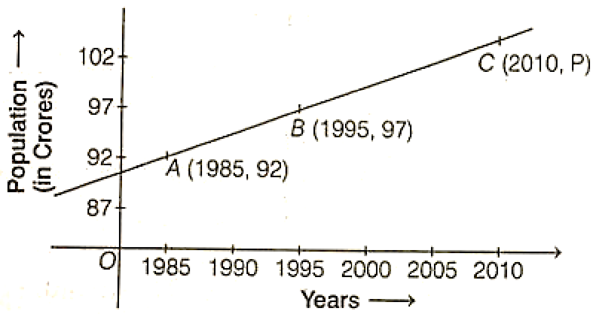
iii. Find the correct mean. (2)

OR

Find the sum of correct scores. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Population vs Year graph given below.



i. In which year the population becomes 110 crores? (1)

ii. Find the equation of line perpendicular to line AB and passing through (1995, 97). (1)

iii. Write the equation of line AB? (2)

OR

Find the slope of line AB. (2)

Solution

Section A

1.

(d) $\frac{-1}{5}$

Explanation:

In quadrant III, sin θ < 0.

$\sin^2\theta = \left(1-\cos^2\theta\right) = \left(1-\frac{144}{169}\right) = \frac{25}{169} \Rightarrow \sin\theta = -\sqrt{\frac{25}{169}} = \frac{-5}{13}$

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-12}{13} \times \frac{13}{-5} = \frac{12}{5}$

$\operatorname{cosec}^2\theta = \left(1+\cot^2\theta\right) = \left(1+\frac{144}{25}\right) = \frac{169}{25} \Rightarrow \operatorname{cosec}\theta = -\sqrt{\frac{169}{25}} = \frac{-13}{5}$

$\therefore \quad (\cot\theta + \operatorname{cosec}\theta) = \left(\frac{12}{5} - \frac{13}{5}\right) = \frac{-1}{5}$

2.

(c) $R \subseteq A \times B$

Explanation:

Let A and B be two sets. Then a relation R from set A to set B is a subset of A×B.Thus, R is a relation from A to B $\Leftrightarrow R \subseteq A \times B$.

3.

(c) $\frac{4}{13}$

Explanation:

If A and B denote the events of drawing a king and a spade card, respectively, then event A consists of four sample points, whereas event B consists of 13 sample points.

Thus, P(A) = $\frac{4}{52}$ and P(B) = $\frac{13}{52}$

The compound event (A ∩ B) consists of only one sample point, king of a spade.

So, P(A ∩ B) = 152

By addition theorem , we get

P (A ∪ B) = P(A) + P (B) − P (A ∩ B)

= $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

Hence, the probability that the card is drawn is either a king or a spade is given by $\frac{4}{13}$

4.

(b) n

Explanation:

$\lim\limits_{x\to 0} \frac{(1+x)^n - 1^n}{(1+x)-(1)} = \underset{x\to 0}{lim}\, n(1+x)^{n-1} = n$

5.

(c) (α, β, -γ)

Explanation:

In xy-plane, the reflection of the point (α, β, γ) is (α, β, -γ)

6. **(a)** 3

Explanation:

We have the general term of $(x + a)^n$ is $T_{r+1} = {}^nC_r(x)^{n-r}a^r$

Now consider $\left(\frac{x}{3} - \frac{2}{x^2}\right)^{10}$

Here $T_{r+1} = {}^{10}C_r \left(\frac{x}{3}\right)^{10-r}\left(-\frac{2}{x^2}\right)^r$

Hence r^{th} term = $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{11-r}\left(-\frac{2}{x^2}\right)^{r-1}$

Comparing the indices of x in x_4 and in T_r, we get

$\Rightarrow$ 11 - r - 2r + 2 = 4

$\Rightarrow$ 3r = 9

$\Rightarrow$ r = 3

7.

(c) $|z| = \frac{\sqrt{3}}{2}$, arg (z) = $\tan^{-1} \frac{1}{\sqrt{2}}$

Explanation:

$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{6}$

$\Rightarrow z = \frac{1}{\sqrt{2}} + \frac{1}{2}i$

$\Rightarrow |z| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{4}}$

$\Rightarrow |z| = \sqrt{\frac{1}{2} + \frac{1}{4}}$

$\Rightarrow |z| = \sqrt{\frac{3}{4}}$

$\Rightarrow |z| = \frac{\sqrt{3}}{2}$

$\tan\alpha = \left|\frac{\mathrm{Im}(z)}{\mathrm{Re}(z)}\right|$

$= \frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

since, the point z lies in the first quadrant.

Therefore, arg (z) $= \alpha = \tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

8.

(c) none of these

Explanation:

f(x) = $\sqrt{\log(2x - x^2)}$

For f(x) to be defined 2x - x^2 should be positive.

solving inequality,

log (2x - x^2) ≥ 0

$\Rightarrow$ 2x - $x^2 \geq e^0$ (log taken to the opposite side of the equation becomes e)

$\Rightarrow$ x^2 - 2x + 1 ≤ 0

$\Rightarrow$ $(x - 1)^2 \leq 0$

$\Rightarrow$ x ≤ 1

Hence, domain of f(x) is $(-\infty, 1)$

9.

(d) $-$ x $>$ $-$ 5

Explanation:

Given x $<$ 5

Multiplying both sides of the above inequality by -1,we get

$-$ x $>$ $-$ 5 [The sign of the inequality is to be reversed if both sides of an inequality are multiplied by the same negative real number]

10.

(b) $\frac{(2+\sqrt{3})}{2}$

Explanation:

$2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$=\sin\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$ [using 2sinASin B= sin (A + B) + sin (A - B)]

$=\left(\sin\frac{\pi}{2}+\sin\frac{\pi}{3}\right)=\left(1+\frac{\sqrt{3}}{2}\right)=\frac{(2+\sqrt{3})}{2}$

11.

(c) None of these

Explanation:

4 $\notin$ A

{4} $\not\subset$ A

B $\not\subset$ A

Therefore, we can say that none of these options satisfy the given relation.

12.

(b) 126

Explanation:

$S_6 = 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$.

Here we have, a = 2 and r = 2.

$S_n=\frac{a(r^n-1)}{(r-1)}\Rightarrow S_6=\frac{2(2^6-1)}{(2-1)}$ = 2 × (64 - 1) = 2 × 63 = 126.

13. **(a)** 1760 x^{-12}

Explanation:

Therefore,in the expansion of $\left(2x^2+\frac{1}{x^2}\right)^{12}$, we have

$T_{r+1}={}^{12}C_r\cdot\left(2x^2\right)^{12-r}\cdot\left(\frac{1}{x^2}\right)^r$

$\Rightarrow T_{10}=T_{9+1}={}^{12}C_9\cdot\left(2x^2\right)^{(12-9)}\cdot\left(\frac{1}{x^2}\right)^9={}^{12}C_3\cdot\left(2x^2\right)^3\cdot\left(\frac{1}{x^2}\right)^9$

$=\frac{12\times11\times10}{3\times2\times1}\times8x^6\cdot\frac{1}{x^{18}}=\frac{1760}{x^{12}}$ = 1760 x^{-12}

14. **(a)** (6, 8),(8, 10), (10, 12)

Explanation:

Let the consecutive even positive integers be x and x + 2.

By data, x > 5 and x + (x + 2) < 23

Now x + (x + 2) < 23

$\Rightarrow$ 2x + 2 < 23

$\Rightarrow$ 2x < 21

$\Rightarrow$ x < $\frac{21}{2}$ = $10\frac{1}{2}$

So we have the least possible value of x is 6 and the maximum value of x is 10.

Therefore the possible pairs of consecutive even positive integers are (6, 8), (8, 10), (10, 12).

15.

(b) { }

Explanation:

Here value of x is not possible so A is a null set.

16.

(c) $2\sqrt{3}$

Explanation:

tan 75° = tan(45° + 30°

$=\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}+1}{\sqrt{3}-1}\therefore cot75^\circ=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Now $tan75^\circ-cot75^\circ=\frac{\sqrt{3}+1}{\sqrt{3}-1}-\frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{(\sqrt{3}+1)^2-(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\Rightarrow tan75^\circ - cot75^\circ = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

17.

(c) $\left(\frac{5}{169} - \frac{12}{169}i\right)$

Explanation:

z = (3 + 2i)^2 = (9 + 4i^2 + 12i) = (9 - 4 + 12i) = (5 + 120)

$\Rightarrow$ z^{-1} = $\frac{1}{(5+12i)} \times \frac{(5-12i)}{(5-12i)} = \frac{(5-12i)}{(25-144i^2)} = \frac{(5-12i)}{(25+144)} = \frac{(5-12i)}{(169)}$

$\Rightarrow$ z^{-1} = $\left(\frac{5}{169} - \frac{12}{169}i\right)$

18.

(b) 12

Explanation:

All S's can be placed either at even places or at odd places, i.e. in 2 ways.

The remaining letters can be placed at the remaining places in 3!, i.e. in 6 ways.

$\therefore$ Total number of ways = 6 $\times$ 2 = 12

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20.

(c) A is true but R is false.

Explanation:

Assertion Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{4+7+8+9+10+12+13+17}{8}$ = 10

xi	**|xi - $\bar{x}$|**
4	|4 - 10| = 6
7	|7 - 10| = 3
8	|8 - 10| = 2
9	|9 - 10| = 1
10	|10 - 10| = 0
12	|12 - 10| = 2
13	|13 - 10| = 3
17	|17 - 10| = 7
$\sum x_i$ = 80	$\sum \lvert x_i - \bar{x} \rvert$ = 24

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i-\bar{x}|}{n} = \frac{24}{8}$ = 3

Reason Mean of the given series

$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$

$= \frac{38+70+48+40+42+55+63+46+54+44}{}$ = 50

$\therefore$ Mean deviation about mean

$= \frac{\Sigma|x_i-\bar{x}|}{n}$

$= \frac{84}{10}$ = 8.4

Hence, Assertion is true and Reason is false.

Section B

21. We have, R = {(x, y) : x $\in$ N, y$\in$ N and x + y = 10}

$\Rightarrow$ R = {$(1,9),(2,8),(3,7),(4,6),(5,5),(6,4),(7,3),(8,2),(9,1)$ }

$\therefore$ Domain (R) = Set of first elements of the ordered pairs in relation R

= {$1,2,3,4,5,6,7,8,9$ }

Range (R) = Set of second elements of the ordered pairs in relation R

= {$9,8,7,6,5,4,3,2,1$ }

OR

Here we are given that, (a + 3, b - 2) = (5, 1)

$\because$ Given ordered pairs are equal. So, corresponding elements are also equal.

$\therefore$ a + 3 = 5 ....(i) and b - 2 = 1 ....(ii)

After solving eq. (i), we obtain

a + 3 = 5 $\Rightarrow$ a = 2

After solving eq. (ii), we obtain

b - 2 = 1 $\Rightarrow$ b = 3

Therefore, the value of a = 2 and b = 3.

22. We have given that

$\lim_{x\to a}\left[\frac{x^5-a^5}{x-a}\right]$ = 405

$\Rightarrow$ 5a^4 = 405

$\Rightarrow$ a^4 = 81

$\Rightarrow$ a = $\pm$3

23. The equations of the given lines are:

x - y = 4 ...(i)

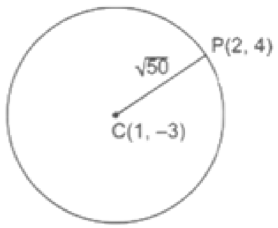
2x + 3y = -7 ...(ii)

Solving equation(i) and equation(ii), we get x =1 and y = - 3.

So, the point of intersection of the given lines is C(1, - 3).

$\therefore$ centre of the given circle is C(1, - 3).

Also, the circle passes through the point P(2, 4).



$\therefore$ radius of the circle

= |CP| = $\sqrt{(1-2)^2+(-3-4)^2}=\sqrt{50}$

$\therefore$ the required equation of the circle is (x + 1)^2 + (y + 3)^2 = $(\sqrt{50})^2$

$\Rightarrow$ x^2 + y^2 - 2x + 6y - 40 = 0

OR

We are given:

y^2 - 4y + 4x = 0

$\Rightarrow$ (y - 2)^2 - 4 + 4x = 0

$\Rightarrow$ (y - 2)^2 = - 4 (x - 1)

Let Y = y - 2

X = x - 1

Then, we have:

Y^2 = - 4X

On comparing the given equation with $Y^2 = -4aX$

$4a = 4 \Rightarrow a = 1$

$\therefore$ Vertex = (X = 0, Y = 0) = (x = 1, y = 2)

Focus = (X = -a, Y = 0) = (x - 1 = - 1, y - 2 = 0) = (x = 0, y = 2)

Equation of the directrix:

x = a

i.e. x - 1 = 1 $\Rightarrow$ x = 2

Axis = Y = 0

i.e. y - 2 = 0 $\Rightarrow$ y = 2

Therefore, length of the latus rectum = 4a = 4 units

24. A = {2, 3} and B = {x : x is solution of $x^2 + 5x + 6 = 0$}

Now $x^2 + 5x + 6 = 0 \Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow (x + 3)(x + 2) = 0 \Rightarrow x = -3, -2$

$\therefore$ B = {-2, -3}

Hence A and B are not equal sets.

25. If m_1 and m_2 are the slopes of a line, tangent of angle between the line is, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Let slope of one line be m, then slope of another line be 2m.

Given, the tangent of the angle between them is $\tan\theta = \frac{1}{3}$

$\therefore \quad \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1}{3} \Rightarrow \frac{1}{3} = \left|\frac{m - 2m}{1 + m \times 2m}\right|$

$\Rightarrow \quad \frac{1}{3} = \left|\frac{-m}{1 + 2m^2}\right|$

$\Rightarrow (1 + 2m^2) = 3m$

$\Rightarrow 2m^2 - 3m + 1 = 0$

Factorise it by splitting the middle term.

$\Rightarrow 2m^2 - 2m - m + 1 = 0$

$\Rightarrow 2m(m - 1) - 1(m - 1) = 0$

$\Rightarrow (2m - 1)(m - 1) = 0$

$\Rightarrow$ 2m - 1 = 0 or m - 1 =0 $\Rightarrow m = \frac{1}{2}, m = 1$

Section C

26. i. Here we have, X = {1, 2, 3, 4} and Y = {1, 4, 9, 16, 25}.

and f = {(x, y): x $\in$ X, y $\in$ Y and $y = x^2$}.

We have, f = {(x, y): x $\in$ X, y $\in$ Y and $y = x^2$}.

Giving different values to x from the set X and getting the corresponding value of $y = x^2$, we get

f = {(1,1), (2,4), (3,9), (4,16)}.

Clearly, every element in X has a unique image in Y.

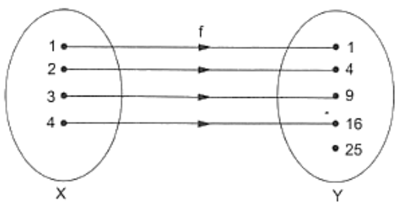
Hence, f is a function from X to Y.

Dom (f) = {1, 2, 3, 4} = X.

Range (f) = {1, 4, 9, 16} $\subset$ Y.

Clearly, 25 $\in$ Y does not have its pre-image in X.

ii. A pictorial representation of the above mapping f is given below.



iii. Now, let A = {2, 3, 4}. Then,

$f(2) = 2^2 = 4$, $f(3) = 3^2 = 9$ and $f(4) = 4^2 = 16$.

$\therefore$ f(A) = {f(x) : x $\in$ A} = {4, 9, 16).

27. Here $\frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

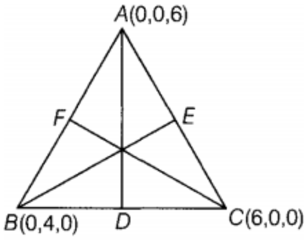
$\Rightarrow \frac{2x}{3} - \frac{1}{3} \geqslant \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5}$

$\Rightarrow \frac{2x}{3}-\frac{3x}{4}-\frac{x}{5} \geqslant \frac{-2}{4}-\frac{2}{5}+\frac{1}{3}$
$\Rightarrow \frac{40x-45x-12x}{60} \geqslant \frac{-30-24+20}{60}$
$\Rightarrow \frac{-17x}{60} \geqslant \frac{-34}{60}$
Multiplying both sides by 60, we have
$-17x \geqslant -34$
Dividing both sides by -17, we have
$\frac{-17x}{-17} \leqslant \frac{-34}{-17}$
$\Rightarrow x \leqslant 2$
Thus the solution set is $(-\infty, 2]$

28. Here, we have to find the number of different signals that can be generated by arranging at least 2 flags in order i.e., a signal can consist of either 2 flags, 3 flags, 4 flags or 5 flags. If a signal consists of 2 flags, then vacant places are 2 and 5 flags are available.
$\therefore$ Number of ways of filling first vacant place = 5
Number of ways of filling second vacant place = 4
Then, the total number of signals consisting by 2 flags
= 5 $\times$ 4 = 20
Now, if a signal consists of 3 flags, then vacant places are 3 and 5 flags are available.
$\therefore$ Number of ways of filling of first vacant place = 5
Number of ways of filling of second vacant place = 4
Number of ways of filling of third vacant place = 3
Then, the total number of signals consisting by 3 flags
= 5 $\times$ 4 $\times$ 3 = 60
Similarly, the total number of signals consisting of 4 flags
= 5 $\times$ 4 $\times$ 3 $\times$ 2 = 120
Total number of signals consisting of 5 flags
= 5 $\times$ 4 $\times$ 3 $\times$ 2 $\times$ 1 = 120
Now, different signals can be generated by arranging either 2 flags or 3 flags or 4 flags or 5 flags. So, by the fundamental principle of addition, we get
Total number of signals = 20 + 60 + 120 + 120 = 320

OR

ABC is a triangle with vertices A (0, 0, 6), (0, 4, 0) and C (6, 0, 0).
Let points D, E and F are the mid-points of BC, AC and AB, respectively. So, AD, BE and CF will be the medians of the triangle.



Coordinates of point $D=\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right)$ = (3, 2, 0)
$\left[\because \text{ coordinates of mid-point } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)\right]$
Coordinates of point $E=\left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right)$ = (3, 0, 3)
and coordinates of point$F=\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right)=(0,2,3)$
Now, length of median
AD = Distance between point A and D
$AD=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}$
[$\because$ distance $=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2+\left(z_1-z_2\right)^2}$]
$=\sqrt{9+4+36}$
$=\sqrt{49}=7$
Similarly, $BE=\sqrt{(0-3)^2+(4-0)^2+(0-3)^2}$
$=\sqrt{9+16+9}=\sqrt{34}$
and $CF=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}$

$= \sqrt{36+4+9} = \sqrt{49} = 7$

Hence, length of the medians are 7, $\sqrt{34}$ and 7.

29. Suppose r^{th}, $(r + 1)^{th}$ and $(r + 2)^{th}$ terms are the three consecutive terms.whose coefficient are 56, 70, 56 respectively.

But Their respective coefficients are ${}^nC_{r-1}$, ${}^nC_r$ and ${}^nC_{r+1}$

Therefore We have:

${}^nC_{r-1} = {}^nC_{r+1} = 56$

$\Rightarrow$ r = 1 + r + 1 = n [If ${}^nC_r = {}^nC_s \Rightarrow$ r = s or r + s = n]

$\Rightarrow$ 2r = n

$\Rightarrow r = \frac{n}{2}$

Now,

${}^nC_{\frac{n}{7}} = 70$ and ${}^nC_{\left(\frac{n}{2}-1\right)} = 56$

$\Rightarrow \frac{{}^nC\left(\frac{n}{2}-1\right)}{nC_{\frac{n}{2}}} = \frac{56}{70}$

$\Rightarrow \frac{\frac{n}{2}}{\left(\frac{n}{2}+1\right)} = \frac{8}{10}$

$\Rightarrow$ 5n = 4n + 8

$\Rightarrow$ n = 8

So, r = $\frac{n}{2}$ = 4

Thus, the required terms are 4th, 5th and 6th.

OR

As discussed in the previous example, the middle term in the expansion of $(1 + x)^{2n}$ is given by $T_{n+1} = {}^{2n}C_n x^n$

So, the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is ${}^{2n}C_n$.

Now, consider the expansion of $(1 + x)^{2n-1}$ Here, the index (2n-1) is odd.

So, $\left(\frac{(2n-1)+1}{2}\right)^{th}$ and $\left(\frac{(2n-1)+1}{2}+1\right)^{th}$ i.e., n^{th} and $(n + 1)^{th}$ terms are middle terms.

Now, $T_n = T_{(n-1)+1}$,= ${}^{2n-1}C_{n-1}(1)^{(2n-1)-(n-1)}x^{n-1}$ = ${}^{2n-1}C_{n-1}x^{n-1}$

and, T_{n+1} = ${}^{2n-1}C_n(1)^{(2n-1)-n}x^n = {}^{2n-1}C_n x^n$

So, the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$ are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$.

$\therefore$ Sum of these coefficients = ${}^{2n-1}C_{n-1} + {}^{2n-1}C_n$

= ${}^{(2n-1)+1}C_n$ [$\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$]

= ${}^{2n}C_n$

= Coefficient of middle term in the expansion of $(1 + x)^{2n}$.

30. Let $z = \frac{1}{(1+i)} \times \frac{(1-i)}{(1-i)} = \frac{(1-i)}{(1-i^2)} = \frac{(1-i)}{2} = \left(\frac{1}{2} - \frac{1}{2}i\right)$

Let its polar form be $z = r(\cos\theta + i\sin\theta)$ ...(1)

Now, $r = |z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

Let α be the acute angle, given by

$\tan\alpha = \left|\frac{\text{Im}(z)}{\text{Re}(z)}\right| = \left|\frac{(-1/2)}{(1/2)}\right| = 1 \Rightarrow \alpha = \frac{\pi}{4}$

Clearly, the point representing $z = \left(\frac{1}{2} - \frac{1}{2}i\right)$ is $P\left(\frac{1}{2}, \frac{-1}{2}\right)$, which lies in the fourth quadrant.

argetz) $= \theta = -a = -\frac{\pi}{4}$

Thus, $r = |z| = \frac{1}{\sqrt{2}}$ and $\theta = \frac{-\pi}{4}$

put value of r and θ in equation (1)

$z = \frac{1}{\sqrt{2}}\left\{\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right\}$,

Hence, the required polar form is $\frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$

OR

Here $3x^2 - 4x + \frac{20}{3} = 0$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$, we have a = 3, b = -4 and $c = \frac{20}{3}$

$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times \frac{20}{3}}}{2 \times 3} = \frac{4 \pm \sqrt{16-80}}{6}$

$= \frac{4 \pm \sqrt{-64}}{6} = \frac{4 \pm 8\sqrt{-1}}{6} = \frac{4 \pm 8i}{6} = \frac{2 \pm 4i}{3}$

Thus $x = \frac{2+4i}{3}$ and $x = \frac{2-4i}{3}$

31. Here

n(U) = a + b + c + d + e + f + g + h = 60 ....(i)

n(H) = a + b + c +d = 25 ....(ii)

n(T) = b + c + f + g = 26 .....(iii)

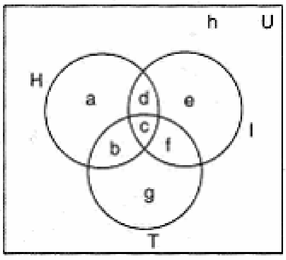
n(I) = c + d + e + f = 26 ....(iv)

$n(H \cap I)$ = c + d = 9 ....(v)

$n(H \cap T)$= b + c = 11 .....(vi)

$n(T \cap I)$ = c + f = 8 .....(vii)

$n(H \cap T \cap I)$ = c = 3 ....(viii)



Putting value of c in (vii),

3 + f = 8 ⇒ f = 5

Putting value of c in (vi),

3 + b = 11 ⇒ b = 8

Putting values of c in (v),

3 + d = 9 ⇒ d = 6

Putting value of c, d, f in (iv),

3 + 6 + e + 5 = 26 ⇒ e = 26 - 14= 12

Putting value of b, c, f in (iii),

8 + 3 + 5 + g = 26 ⇒ g = 26 - 16= 10

Putting value of b, c, d in (ii)

a + 8 + 3 + 6 = 25 ⇒ a = 25 - 17 = 8

Number of people who read exactly one newspapers

= a + e + g

= 8 + 12 + 10 = 30

Section D

32. Given that:Sample Space S = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9}

A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

$P(e_1) = P(e_2) = .08$, $P(e_3) = P(e_4) = P(e_5) = .1$

$P(e_6) = P(e_7) = .2$, $P(e_8) = P(e_9) = .07$

i. To find: P(A), P(B) and P(A ∩ B)

a. A = {e_1, e_5, e_8}

On adding the probabilities of elements of A, we get

$P(A) = P(e_1) + P(e_5) + P(e_8)$

⇒ P(A) = 0.08 + 0.1 + 0.07 [given]

⇒ P(A) = 0.25

b. B = {e_2, e_5, e_8, e_9}

Similarly, on adding the probabilities of elements of B, we get

$P(B) = P(e_2) + P(e_5) + P(e_8) + P(e_9)$

$\Rightarrow$ P(B) = 0.08 + 0.1 + 0.07 + 0.07 [given]

$\Rightarrow$ P(B) = 0.32

c. Now, we have to find P(A ∩ B)

A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

$\therefore$ A ⋂ B = {e_5, e_8}

On adding the probabilities of elements of A ⋂ B, we get

P(A ⋂ B) = P(e_5) + P(e_8)

= 0.1 + 0.07 = 0.17

ii. To find: P(A ∪ B)

a. By General Addition Rule: P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

from part (a), we have

P(A) = 0.25, P(B) = 0.32 and P(A ∩ B) = 0.17

Putting the values, we get

P(A ∪ B) = 0.25 + 0.32 – 0.17 = 0.40

iii. A = {e_1, e_5, e_8} and B = {e_2, e_5, e_8, e_9}

$\therefore$ A ⋃ B = {e_1, e_2, e_5, e_8, e_9}

Again, on adding the probabilities of elements of A ⋃ B, we get

P(A ⋃ B) = P(e_1) + P(e_2) + P(e_5) + P(e_8) + P(e_9)

= 0.08 +0.08 + 0.1 + 0.07 + 0.07 = 0.40

iv. To find: P($\bar{B}$)

a. By Complement Rule, we have

P ($\bar{B}$) = 1 - P(B)

$\Rightarrow$ P ($\bar{B}$) = 1 - 0.32 = 0.68

b. Given: B = {e_2, e_5, e_8, e_9}

$\therefore$ $\bar{B}$ = {e_1, e_3, e_4, e_6, e_7}

P($\bar{B}$) = P(e_1) + P(e_3) + P(e_4) + P(e_4) + P(e_6) + P(e_{17})

= 0.08 + 0.1 + 0.1 + 0.2 + 0.2 [given] = 0.68

33. We have, f(x) = sec x

By using first principle of derivative,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0} \frac{\sec(x+h)-\sec x}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}$$

$$= \lim_{h\to 0} \frac{\cos x - \cos(x+h)}{h \times \cos x \cdot \cos(x+h)}$$

$$= \lim_{h\to 0} \left[\frac{-2\sin\left(\frac{x+x+h}{2}\right)\cdot \sin\frac{(x-x-h)}{2}}{h\cdot \cos x \cdot \cos(x+h)}\right]$$

$$\left[\because \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$$

$$= \lim_{h\to 0} \left[\frac{-2\sin\left(x+\frac{h}{2}\right)\cdot\left(-\sin\frac{h}{2}\right)}{h\cdot \cos x \cos(x+h)}\right]$$

$$= \lim_{h\to 0} \frac{\sin\left(x+\frac{h}{2}\right)}{\cos(x+h)\cdot \cos x} \cdot \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= \frac{\sin x}{\cos^2 x} \times (1) = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

= tanx × secx

OR

We have, f(x) = sinx + cosx

By using first principle of derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$\therefore f'(x)=\lim_{h\to 0}\frac{\sin(x+h)+\cos(x+h)-\sin x-\cos x}{h}$

$=\lim_{h\to 0}\frac{[\sin x\cdot\cos h+\cos x\cdot\sin h+\cos x\cdot\cos h-\sin x\cdot\sin h-\sin x-\cos x]}{h}$ [∵ sin (x + y) = sin x cos y + cos x sin y and cos (x + y) = cos x cos y - sin x sin y]

$=\lim_{h\to 0}\frac{[(\cos x\cdot\sin h-\sin x\cdot\sin h)+(\sin x\cdot\cos h-\sin x)+(cosx\cdot cosh-cosx)]}{h}$

$=\lim_{h\to 0}\frac{\sin h(\cos x-\sin x)+\sin x(\cos h-1)+\cos x(\cos h-1)}{h}$

$=\lim_{h\to 0}\frac{\sin h}{h}(\cos x-\sin x)+\lim_{h\to 0}\frac{\sin x(\cos h-1)}{h}+\lim_{h\to 0}\frac{\cos x(\cos h-1)}{h}$

$=1\cdot(\cos x-\sin x)+\lim_{h\to 0}\sin x\left[\frac{-(1-\cos h)}{h}\right]+\lim_{h\to 0}\cos x\left[\frac{-(1-\cos h)}{h}\right]\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$

$=(\cos x-\sin x)-\sin x\cdot\lim_{h\to 0}\left(\frac{1-\cos h}{h}\right)-\cos x\cdot\lim_{h\to 0}\left(\frac{1-\cos h}{h}\right)$

$=(\cos x-\sin x)-\sin x\cdot\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h\times\frac{h}{4}}\times\frac{h}{4}-\cos x\cdot\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h\times\frac{h}{4}}\times\frac{h}{4}$

$=(\cos x-\sin x)-\sin x\cdot 2\cdot\frac{1}{4}\lim_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times h-\cos x\cdot 2\cdot\frac{1}{4}\lim_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 h$

$=(\cos x-\sin x)-\frac{1}{2}\cdot\sin x\cdot(1)\times 0-\cos x\cdot\frac{1}{2}\cdot(1)\times 0\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1\right]$

= (cos x - sin x) - 0 - 0

= cos x - sin x

34. $\frac{\frac{a+b}{2}}{\sqrt{ab}}=\frac{m}{n}$

$\frac{a+b}{2\sqrt{ab}}=\frac{m}{n}$

by C and D

$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{m+n}{m-n}$

$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{m+n}{m-n}$

$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{m+n}}{\sqrt{m-n}}$

by C and D

$\frac{\sqrt{a}}{\sqrt{b}}=\frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$

Sq both side

$\frac{a}{b}=\frac{m+n+m-n+2\sqrt{m^2-n^2}}{m+n+m-n-2\sqrt{m^2-n^2}}$

$\frac{a}{b}=\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

35. **LHS** $=cos12^o+cos60^o+cos84^o$

$=cos12^o+(cos84^o+cos60^o)$

$=cos12^o+[2cos\left(\frac{84^\circ+60^\circ}{2}\right)\times cos\left(\frac{84^\circ-60^\circ}{2}\right)]$

[∵ cosx +cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$=cos12^o+[2cos\,\frac{144^\circ}{2}\times\cos\frac{24^\circ}{2}]$

$=cos12^o+[2cos72^o\times cos12^o]=cos12^o[1+2cos72^o]$

$=cos12^o[1+2cos(90^o-18^o)]$

= cos 12^o [1 + 2 sin 18^o] [∵ cos (90^o - θ) = sin θ]

= cos 12^o [1 + 2 $\left(\frac{\sqrt{5}-1}{4}\right)$] [∵ sin 18^o = $\frac{\sqrt{5}-1}{4}$]

= $\left(1+\frac{\sqrt{5}-1}{2}\right)$ cos 12^o = $\left(\frac{\sqrt{5}+1}{2}\right)$ cos 12^o

RHS = cos 24^o + cos 48^o

= 2 cos $\left(\frac{24^\circ+48^\circ}{2}\right)$ cos $\left(\frac{24^\circ-48^\circ}{2}\right)$ [∵ cosx + cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$=2cos36^o cos(-12^o)$

= 2 cos 36^o × cos 12^o [∵ cos (- θ) = cos θ]

= 2 × $\frac{\sqrt{5}+1}{4}$ × cos 12° = $\frac{\sqrt{5}+1}{2}$ × cos 12° [∵ cos 36° = $\frac{\sqrt{5}+1}{4}$]

∴ **LHS = RHS**

Hence proved.

OR

Given, $sin\,\theta + sin\,\phi = \sqrt{3}\,(cos\,\phi - cos\,\theta)$

$\therefore 2sin\left(\frac{\theta+\phi}{2}\right) \times cos\left(\frac{\theta-\phi}{2}\right) = \sqrt{3}\left[-2sin\left(\frac{\phi+\theta}{2}\right) \times sin\left(\frac{\phi-\theta}{2}\right)\right]$

$\Rightarrow 2sin\left(\frac{\theta+\phi}{2}\right) \times cos\left(\frac{\theta-\phi}{2}\right) = \sqrt{3}\left[-2sin\left(\frac{\theta+\phi}{2}\right) \times sin\left\{-\left(\frac{\theta-\phi}{2}\right)\right\}\right]$

$\Rightarrow 2sin\left(\frac{\theta+\phi}{2}\right) \times cos\left(\frac{\theta-\phi}{2}\right) = \sqrt{3}\left[2sin\left(\frac{\theta+\phi}{2}\right) \times sin\left(\frac{\theta-\phi}{2}\right)\right]$

$\Rightarrow 2sin\left(\frac{\theta+\phi}{2}\right) \times cos\left(\frac{\theta-\phi}{2}\right) - 2\sqrt{3}\,sin\left(\frac{\theta+\phi}{2}\right) \times \sin\left(\frac{\theta-\phi}{2}\right) = 0$

$\Rightarrow 2\sin\left(\frac{\theta+\phi}{2}\right)\left[\cos\left(\frac{\theta-\phi}{2}\right) - \sqrt{3}\sin\left(\frac{\theta-\phi}{2}\right)\right] = 0$

$\Rightarrow \sin\left(\frac{\theta+\phi}{2}\right)\left[\cos\left(\frac{\theta-\phi}{2}\right) - \sqrt{3}\sin\left(\frac{\theta-\phi}{2}\right)\right] = 0$

If $\sin\left(\frac{\theta+\phi}{2}\right) = 0$, then $\sin\left(\frac{\theta+\phi}{2}\right) = \sin 0$

$\Rightarrow \left(\frac{\theta+\phi}{2}\right) = 0 \Rightarrow \theta + \phi = 0$

$\Rightarrow \theta = -\phi$

∴ LHS = sin 3θ + sin 3ϕ

= sin (- 3ϕ) + sin 3ϕ

= - sin 3ϕ + sin 3ϕ = 0

= RHS

and if $\left[\cos\left(\frac{\theta-\phi}{2}\right) - \sqrt{3}\sin\left(\frac{\theta-\phi}{2}\right)\right] = 0$

Then, $\cos\left(\frac{\theta-\phi}{2}\right) = \sqrt{3}\sin\left(\frac{\theta-\phi}{2}\right)$

$\therefore \tan\left(\frac{\theta-\phi}{2}\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\left(\frac{\theta-\phi}{2}\right) = \tan\frac{\pi}{6}$

$\therefore \left(\frac{\theta-\phi}{2}\right) = \frac{\pi}{6} \Rightarrow \theta - \phi = \frac{2\pi}{6} = \frac{\pi}{3}$

$\Rightarrow 3\theta - 3\phi = \pi \Rightarrow 3\theta = \pi + 3\phi$

∴ **LHS** $= sin3\theta + sin3\phi$

= sin (π + 3ϕ) + sin 3ϕ [put 3θ = π + 3ϕ]

= - sin 3ϕ + sin 3ϕ = 0 **= RHS**

∴ **LHS = RHS**

Hence proved.

Section E

36. i. An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant. Hence path traced by Arun is ellipse.
    Sum of the distances of the point moving point to the foci is equal to length of major axis =10m
  ii. Given 2a = 10 & 2c = 8
    $\Rightarrow$ a = 5 & c = 4
    $c^2 = a^2 + b^2$
    $\Rightarrow 16 = 25 + b^2$
    $\Rightarrow b^2 = 25 - 16 = 9$
    Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
    Required equation is $\frac{x^2}{25} + \frac{y^2}{9} = 1$
  iii. equation is of given curve is $\frac{x^2}{25} + \frac{y^2}{9} = 1$
    a = 5, b = 3 and given 2c = 8 hence c = 4
    Eccentricity = $\frac{\text{c}}{\text{a}} = \frac{4}{5}$
    OR
    $\frac{x^2}{25} + \frac{y^2}{9} = 1$

Hence a = 5 and b = 3

Length of latus rectum of ellipse is given by $\frac{2b^2}{a} = \frac{2\times9}{5} = \frac{18}{5}$

37. i. SD = σ = 15

$\Rightarrow$ Variance = 15^2 = 225

According to the formula,

Variance = $\left(\frac{1}{n}\sum {x_\mathrm{i}}^2\right) - \left(\frac{1}{n}\sum \mathrm{x_i}\right)^2$

$\therefore \frac{1}{200}\sum x_\mathrm{i}^2 - (40)^2$ = 225

$\Rightarrow \frac{1}{200}\sum (x_\mathrm{i})^2 - 1600$ = 225

$\Rightarrow \sum (x_\mathrm{i})^2$ = 200 $\times$ 1825 = 365000

This is an incorrect reading.

$\therefore$ Corrected $\sum (x_i)^2$ = 365000 - 34^2 - 53^2 + 43^2 + 35^2

= 365000 - 1156 - 2809 + 1849 + 1225

= 364109

Corrected variance = $\left(\frac{1}{n} \times \text{ Corrected } \sum x_i\right)$ - (Corrected mean)^2

= $\left(\frac{1}{200} \times 364109\right) - (39.955)^2$

= 1820.545 - 1596.402

= 224.14

ii. The formula of variance is $\frac{\sum\limits_{i=1}^{n}(x_i-\bar{x})^2}{n}$.

iii. Corrected mean = $\frac{\text{Corrected } \sum x_1}{200}$

= $\frac{7993}{200}$

= 39.955

OR

We have:

n = 200, $\bar{X}$ = 40, σ = 15

$\frac{1}{n}\sum x_\mathrm{i} = \bar{X}$

$\therefore \frac{1}{200}\sum x_i$ = 40

$\Rightarrow \sum x_i$ = 40 $\times$ 200 = 8000

Since the score was misread, this sum is incorrect.

$\Rightarrow$ Corrected $\sum x_i$ = 8000 - 34 - 53 + 43 + 35

= 8000 - 7

= 7993

38. i. Equation of line AB is,

x - 2y = 1801

Putting y = 110,

$\therefore$ x = 1801 + 220

$\Rightarrow$ x = 2021

ii. $\because$ Slope of AB = $\frac{1}{2}$

Slope of the perpendicular of AB = $\frac{-1}{\frac{1}{2}}$ = -2

$\therefore$ Equation of line perpendicular to AB passing through (1995, 97) is

$\Rightarrow$ y - 97 = -2(x - 1995)

$\Rightarrow$ y - 97 = -2x + 3990

$\Rightarrow$ 2x + y = 4087

iii. Equation of line AB is,

$\Rightarrow$ y - y_1 = m(x - x_1)

$\therefore$ y - 92 = $\frac{1}{2}$(x - 1985)

2y - 184 = x - 1985

$\Rightarrow$ x - 2y = 1801

OR

Slope of line AB joining points A(1985, 92) and B(1995, 97)

m = $\frac{97-92}{1995-1985}$ = $\frac{5}{10} = \frac{1}{2}$